RELIABILITY EVALUATION AND COMPONENT ALLOCATION IN SERIES AND PARALLEL SYSTEMS CONSISTING OF NON-IDENTICAL THREE-STATE COMPONENTS

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Systems composed of non-identical multi-state components require extensive combinatorial procedures for reliability evaluation. As a consequence, there is no clear understanding of how reliability is affected when such components are systematically added to the system. The simplified approach presented here aims to evaluate reliability variations of series and parallel systems when non-identical three-state components are considered. Additionally, a nonlinear optimization model for allocation of these components under financial and physical restrictions is suggested. Simulations conducted to robust and stable allocations when distinct magnitudes of failure probabilities were tested.

Palavras-chaves: Multi-state systems, non-identical components, optimal component allocation
1. Introduction

Failures of components and systems can lead to severe consequences, many times causing causalities and large material losses. In order to better understand the way these failures happen and to prevent such occurrences, several reliability studies have been performed in distinct areas to project, develop and maintain components to perform satisfactorily during a defined period of time. Among these reliability issues, Multi-state System Reliability has received crescent attention. Multi-state systems consist of units presenting one working state and two or more failed states with distinct failure modes. In the case of two failure modes (called three-state system), such modes are usually defined as open and short.

Reliability evaluation and optimization of systems consisting of three-state components with identical probabilities of failure can be straightforwardly performed. However, this is not the case of three-state systems with distinct probabilities of open and short failure modes, which require the description of all possible scenarios that conduct the system to failure. This procedure generates a highly combinatorial problem, especially when considering complex systems composed of an elevated number of components. As a consequence, the way that distinct magnitudes of failure probabilities affect the system reliability is not well understood.

This paper addresses this problem by identifying mathematical patterns presented by simple structures (series and parallel systems). Equations for reliability estimation are first developed for a small number of components and thereafter expanded to systems with a larger number of components. Distinct open and short failure probabilities, as well as distinct number of components, are analyzed in terms of their impact over the reliability profiles. In addition, a nonlinear optimization programming model is proposed for the best allocation of non-identical components in scenarios subjected to constraints in budget and number of components.

The remaining of this paper is organized as follows. Section 2 gives a basic description of reliability literature focused on three-state components systems, while Section 3 presents the proposed approach. Numerical examples are depicted in Section 4, and a brief conclusion is presented in Section 5.

2. Background

Reliability is typically defined as the probability that a system, product or component will perform its designed function during a certain period of time, when operating under stated environmental conditions. A vast literature focused on reliability concepts and estimation (KAPUR; LAMBERSON, 1977; O’CONNOR, 1985; ELSAYED, 2006), optimization, allocation and redundancy (COIT; SMITH, 1995; MAJETY et al., 1999; KUO, 2000; ELEGBEDE; CHU, 2003; RAMIREZ-MARQUEZ et al., 2004a, 2008) has emerged in the last years. A more recent problem, multi-state systems have also been subject of intense studies (LISNIANSKI; LEVITIN, 2003; ROCCO; MUSELLI, 2005; RAMIREZ-MARQUEZ; COIT, 2004b, 2005; RAMIREZ-MARQUEZ et al., 2006a, 2006b).

A multi-state device can present two or more failure states, in addition to its normal functioning. Devices with specifically two failures modes are defined as three-state components, often with open and short failure modes. This terminology is originated from
electrical circuits, typically represented by diodes and rheostats. A diode intends to allow the flow of current in one direction and block it from coming back in an opposite direction. Such device can either operate properly, or fail by blocking the current in forward direction, or fail by permitting the current to go back (Page; Perry, 1988; Elsayed, 2006).

Reliability evaluation of systems composed of three-state components is conceptually more complex than systems with binary-state components, even when considering identical units. Major developments performed on this field seem to be focused on (i) the proposition of methods to evaluate the reliability of systems composed of identical and non-identical components, and (ii) on the optimization of a system reliability subjected to structural constraints (Levitin; Lisnianski, 2001).


The optimization of systems composed of multi-state components portrays the second major area of work, being relevant in scenarios where a required level of reliability has to be achieved under certain constraints, usually related to cost or physical issues. Series-parallel systems of identical components were optimized by Page and Perry (1988) through the development of an open-mode reliability polynomial, which tests several alternative configurations for equivalence and identifies the best structure to be adopted. However, such approach is not applicable on systems composed of non-identical components.

Levitin and Lisnianski (2001) proposed a method to optimize multi-state systems based on the combination of a universal generating function (UGF) and genetic algorithm (GA). This methodology efficiently estimates the reliability of series-parallel systems with different failure probabilities and can be applied in several types of optimization problems, such as structure optimization, optimal expansion of an existing system and maintenance optimization. Also using GA techniques, Levitin (2002a) has suggested an algorithm to identify optimal structure of systems composed of devices with distinct open and short failure probabilities. The structure considered in the approach followed a series-parallel configuration. In addition, an optimization heuristic for maximizing the reliability of multi-state weighted voting systems based on a defined number of components with known reliability properties was presented by Ramirez-Marquez (2008).

Multi-state structures following the $k$-out-of-$r$-from-$n$:F configurations were analyzed by Levitin (2002b; 2003), under a “sliding-window” approach. Such a methodology can be briefly explained as follows: consider $n$ ordered multi-state elements with different failure modes and with a performance rate associated with each mode. The system is said to fail whether the sum of rates of consecutive elements is lower than a threshold. The sliding-window scans the order of elements, returning the one with the highest reliability. The
optimization tool utilized in this methodology was the genetic algorithm.

Focused on systems where the use of redundancy is necessary, Levetin et al. (1998) proposed an optimization approach based on the integration of UGF for the fast evaluation of multi-state system reliability and GA as an optimization tool. The redundancy in multi-state systems, however, does not strictly follow the traditional rule of adding several components in parallel in order to achieve the desired level of reliability. Actually, there is an ideal number of redundant components that leads to the maximum reliability, and the addition of extras units is worthless. Expressions for estimating the ideal number of identical redundant components for distinct structures can be obtained in Elsayed (2006). An interesting heuristic for redundancy allocation of multi-state components under a series-parallel structure was also proposed by Ramirez-Marquez and Coit (2004b).

3. Suggested approach

The approach proposed here to evaluate the reliability of non-identical three-state components is structurally simple and can be described in the following steps: (i) analysis of the mathematical similarities among expressions to estimate the reliability of systems composed of a small number of components; (ii) expansion of such expressions from simple structures to larger ones, and analysis of the effect of distinct open and short failure probabilities on reliability patterns under the assumption of increasing number of components; and (iii) application of a nonlinear programming model aiming to optimize the allocation of non-identical components under project restrictions. These steps are better explained in following sections.

It is important to mention that the approach is described based on a series structure; the parallel configuration can be easily derived by switching open and short failure probabilities in the generated expressions.

3.1 Identification of reliability patterns for series systems composed of a small number of non-identical components

This step initially describes the modes under which a series system composed of few components fails. For a system consisting of 2 components (labeled as 1 and 2), the following events conduct the system to failure: (i) component 1 fails open; or (ii) component 2 fails open; or (iii) both components fail short. The reliability expression listing such events is expressed in equation (1).

\[ R_2 = 1 - P(\overline{x}_{1o} + \overline{x}_{2o} + \overline{x}_{1s} + \overline{x}_{2s}) \]  

(1)

where \( \overline{x}_{io} \) and \( \overline{x}_{is} \) denote the probability of component \( i \) to fail in open or short mode, respectively, and \( R \) is the system reliability. Equation (1) can be mathematically expanded, and terms \( \overline{x}_{io} \) and \( \overline{x}_{is} \) substituted by \( q_{io} \) and \( q_{is} \), respectively, for convenience. In addition, \( p_1 + q_{io} + q_{is} = 1 \), where \( p_1 \) is the probability of component \( i \) work properly. The result of these manipulations is shown in equation (2), which is depicted in two lines to make the identification of patterns easier.
In case of having three components in series, possible failure modes are described as performed in previous case. Appropriate manipulations and substitutions lead to equation (3).

\[
R_3 = 1 - q_{1o} - q_{2o} - q_{3o} - q_{1o}q_{2o}q_{3o} + q_{1o}q_{2o} + q_{1o}q_{3o} + q_{2o}q_{3o} - q_{1o}q_{2o}q_{3o}
\]  

Similarly, equation (4) depicts the reliability expression for a system consisting of four components in series.

\[
R_4 = 1 - q_{1o} - q_{2o} - q_{3o} - q_{4o} - q_{1o}q_{2o}q_{3o}q_{4o} + q_{1o}q_{2o} + q_{1o}q_{3o} + q_{2o}q_{3o} + q_{2o}q_{4o} + q_{3o}q_{4o} - q_{1o}q_{2o}q_{3o}q_{4o} + q_{1o}q_{2o}q_{3o}q_{4o}
\]  

Equations (2), (3) and (4) shows that each additional component affects the reliability expression by adding one last row describing the open failure of all \( n \) components and by elevating the number of combinations in each previous row according to \( \binom{n}{m} \), where \( n \) is the number of total components and \( m \) is the number of failed components in each row, from 1 to \( n \). Note that the reliability equation presents a single term describing the short failure mode, indicating that the system only can fail in such mode if all \( n \) components fail. For convenience, this term is brought on the first line of the reliability equation.

The set of equations for a parallel system can be similarly developed, switching positions of \( q_{io} \) and \( q_{jo} \) in above equations.

3.2. Expansion of small structures into larger structures

A simple approach here is to construct sequential spreadsheet tables performing all the necessary combinations of components and respective probabilities, following the pattern exemplified in equations (2) to (4) regarding systems composed of 2, 3, and 4 components in series, respectively. Such combinations are easily implemented in a spreadsheet environment, since the next combination level (due to the addition of next component) utilizes the numerical values calculated on the previous combination (previous table) for its estimation. Basic logical commands and functions for concatenation are utilized for merging the tables and spreading the formulation to subsequent tables. Alternative computational tools could be used for performing this step.

Every level of combination (e.g., \( q_{io}, q_{io}q_{jo}, q_{io}q_{jo}q_{ko}, \ldots \)) is to be obtained in a separated table which generates a positive or negative numerical value according to the level of that combination. From equation (4), it can be noticed that combinations based on an even number
of components (e.g., $q_{io}q_{jo}$) generate positive values, while odd combinations (e.g., $q_{io}$, $q_{io}q_{jo}q_{ko}$) produce negative values. The final reliability ($R$) is obtained by summing the numerical values of all generated combinations to the unitary value.

This step is concluded by analyzing the effect of distinct ratios of open and short failure probabilities ($q_{o}/q_{s}$) on the reliability profiles, under the assumption of increasing number of components. For the proposed analysis, $q_{io}$ and $q_{o}/q_{s}$ are to be defined by the user and $q_{is}$ appears as function of these variables.

3.3 Nonlinear optimization model for allocation of non-identical components under restrictions

The purpose here is to maximize the reliability equation generated in Section 3.2, considering we have $n$ alternative multi-state components with distinct failure probabilities and costs to be assembled in a series system with $k$ components ($k < n$). The task is to choose these $k$ components respecting restrictions of budget and number of units to be used. A closed expression of the reliability function to be optimized for a series structure (which indeed summarizes the tables generated in Section 3.2) is presented in equation (5), where $q_{io}$ and $q_{is}$ denote the probability of component $i$ to fail in open or short mode, respectively.

A binary decision variable $x_i$ is introduced to identify the components to be chosen, returning 1 when component $i$ is to be used or 0 otherwise. Note that variable $x_i$ is present in all combinations terms generated by $n$ candidate components. For modeling a parallel system, $q_{io}$ and $q_{is}$ switch positions in equation (5).

$$\max \prod_{i=1}^{n}(1-x_i q_{io}) - \prod_{i=1}^{n} q_{is}^{x_i}$$

(5)

When using a spreadsheet with an optimization tool for solving the problem, it is necessary to manually link the combinations cells to the respective binary variables. Using the first two levels of probability combinations (i.e., $q_{io}$ and $q_{io}q_{jo}$) is a good alternative when dealing with optimization tools that have restricted number of decision variables. This is a plausible assumption, since elevated levels of combinations are generated by small numbers which will be multiplied by even smaller ones. However, such approximation does not ensure optimal allocation.

The total cost of the system to be assembled is subjected to a budget constraint, as expressed in equation (6).

$$\sum_{i=1}^{n} x_i c_i \leq C$$

(6)

where $x_i$ is the binary variable indicating if component $i$ is included in the system, $c_i$ is the cost of component $i$ and $C$ is the total available budget.

The number of components in the final system is subjected to the constraint in equation (7).

$$\sum_{i=1}^{n} x_i = k$$

(7)
where \( x_i \) keeps the same meaning of previous formulations and \( k \) is the total number of components to appear in the final system.

For purposes of exemplification, Figure 1 brings the final interface of this model implemented in a spreadsheet considering a scenario with 8 candidate components to be assembled in a series structure (What’s Best 8.0 was used as optimization tool). Failure probabilities and cost of each candidate component are depicted, as well as the desired number of components in the final system, maximum budget and \( q_o/q_s \). The output identifies the components to be assembled (1=in, 0=out), as well as the reliability and cost of such a system. In the example of Figure 1, the optimal solution when combining 3 components in series considering a budget limitation of $23 is to allocate components 1, 6 and 8, leading to a reliability of 0.996 and cost of $23.

![Figure 1 - Interface of What’s Best programming](image)

### 4. Numerical examples

Probabilities of open failure of 10 components were randomly generated in the interval 0.008 to 0.09 (see Table 1) and plugged in the tables of the formulation described in Section 3.2. Components included in the analysis followed the order they were labeled (i.e., an analysis of 3 components is based on units 1, 2 and 3), but their order is easily modified by replacing the initial probabilities. Table 2 brings the variation on reliability levels when considering distinct values of \( q_o/q_s \) and under the assumption of increasing the number of components up to 10 units. In this scenario, \( q_{is} \) is a function of \( q_{io} \) and \( q_i/q_s \).

<table>
<thead>
<tr>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{io} )</td>
<td>0.01</td>
<td>0.01</td>
<td>0.05</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
<td>0.008</td>
<td>0.07</td>
<td>0.007</td>
</tr>
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</table>

Fonte: Autores

<table>
<thead>
<tr>
<th>( q_i/q_s )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9701</td>
<td>0.9261</td>
<td>0.9017</td>
<td>0.8573</td>
<td>0.7974</td>
<td>0.7257</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9776</td>
<td>0.9305</td>
<td>0.9031</td>
<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
<tr>
<td>0.4</td>
<td>0.9795</td>
<td>0.9310</td>
<td>0.9032</td>
<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
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<td>0.6694</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9799</td>
<td>0.9311</td>
<td>0.9032</td>
<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
<tr>
<td>1</td>
<td>0.9800</td>
<td>0.9311</td>
<td>0.9032</td>
<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
<tr>
<td>2</td>
<td>0.9801</td>
<td>0.9311</td>
<td>0.9032</td>
<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
</tbody>
</table>

Table 1 - Open failure probabilities for 10 components
The addition of three-state components decreases the reliability of the system (as depicted in Table 2) according to the normal behavior of a series system when extra units are included. However, the decreasing rate suffers smaller reductions as additional components are considered, and the reliability tends asymptotically to 0.66 for structures with 10 or more components. This is due to (i) the magnitude of \( q_o \) values considered in this analysis (different values would lead to different final reliabilities), and (ii) to the fact that \( q_o \)’s are the key terms for explaining variations on the reliability expression of a series system, but only the values of \( q_s \) are subjected to changes under the way this analysis was structured.

Figure 2 brings the reliability profiles with the addition of new non-identical components for 3 values of \( q_o/q_s \). Such profiles are overlapped, showing that distinct \( q_o/q_s \) do not lead to substantial alterations on the reliability pattern.

Table 3 provides the results of a similar analysis performed for a parallel system based on the same values of \( q_o \) and \( q_o/q_s \). In opposition to the series system, it can be noticed that \( q_o/q_s \) significantly affects the reliability of the parallel configuration, especially when the number of components is elevated. The strong effect of \( q_o/q_s \) is justified by the influence of \( q_s \) on the reliability expression of parallel systems. Similarly to the series structure previously presented, the addition of three-state components does not represent elevation of reliability level, what does differ from the normal behavior of parallel systems composed of two-state components when extra units are added. For those, an addition of redundant units leads to reliability elevation in any circumstance (unless components with lower reliabilities are added).
The integration of productive chain with an approach to sustainable manufacturing.

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<table>
<thead>
<tr>
<th>$q_o/q_s$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.8099</td>
<td>0.4050</td>
<td>0.2835</td>
<td>0.1417</td>
<td>0.0425</td>
<td>0.0010</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9024</td>
<td>0.6769</td>
<td>0.5753</td>
<td>0.4315</td>
<td>0.2805</td>
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<td>0.1480</td>
<td>0.0960</td>
<td>0.0950</td>
</tr>
<tr>
<td>0.4</td>
<td>0.9505</td>
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<td>0.7694</td>
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<td>0.5554</td>
<td>0.4304</td>
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<td>0.3474</td>
</tr>
<tr>
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<td>0.8249</td>
<td>0.7527</td>
<td>0.6681</td>
<td>0.6614</td>
<td>0.6035</td>
<td>0.6030</td>
</tr>
<tr>
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<td>0.9800</td>
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<td>0.8580</td>
<td>0.7979</td>
<td>0.7261</td>
<td>0.7203</td>
<td>0.6699</td>
<td>0.6694</td>
</tr>
<tr>
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<td>0.8211</td>
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</tr>
<tr>
<td>5</td>
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</tr>
<tr>
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<td>0.9710</td>
<td>0.9593</td>
<td>0.9583</td>
<td>0.9494</td>
<td>0.9493</td>
</tr>
<tr>
<td>10</td>
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<td>0.9851</td>
<td>0.9782</td>
<td>0.9694</td>
<td>0.9686</td>
<td>0.9618</td>
<td>0.9618</td>
</tr>
</tbody>
</table>

Table 3 - Reliability of a parallel system for distinct levels of $q_o/q_s$ and number of components

Figure 3 shows reliability patterns with the addition of new non-identical components for the same $q_o/q_s$ values used in the previous case. Such profiles present distinct behaviors and show the higher the relation $q_o/q_s$, the smaller the variation over the reliability.

In order to evaluate the effect of the magnitude of failure probabilities over the reliability profiles, Table 4 brings a new set of $q_{io}$ (with higher failure probabilities than those in Table 1), randomly generated in the interval 0.09 to 0.25. A smaller range of $q_o/q_s$ values had to be adopted here to comply with $p_i + q_{io} + q_{is} = 1$, where $p_i$ is the probability of component $i$ work properly.

<table>
<thead>
<tr>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{io}$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.15</td>
<td>0.25</td>
<td>0.22</td>
<td>0.14</td>
<td>0.13</td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 4 - Second set of open failure probabilities ($q_{io}$)

An analysis based on these new values was performed following the approach of previous
examples, and results for the series system are presented in Figure 4. Reliability profiles depicted a slightly different profile for \( q_o/q_s = 0.5 \) when 2 components were considered, what does differ from the identical patterns generated in Figure 2. For remaining scenarios, the effect of distinct \( q_o/q_s \) does not lead to significant differences.

![Figure 4 - Reliability of a series system under distinct \( q_o/q_s \) values and increasing number of components for the second set of \( q_{io} \)](image1)

For a parallel structure, distinct values of \( q_o/q_s \) produced significantly distinct profiles, as depicted in Figure 5. At this point, it is important to mention that, were the analysis structured based on \( q_s/q_o \) instead of \( q_o/q_s \), results regarding both series and parallel structures would have presented a contrary behavior. Such analysis was not performed to avoid repetition of procedures.

![Figure 5 - Reliability of parallel system under distinct levels of \( q_o/q_s \) and increasing number of components for the second set of \( q_{io} \)](image2)
In order to complete the numerical examples, some scenarios regarding the optimal allocation of three-state components with non-identical failure probabilities are presented, based on the nonlinear formulation described in Section 3.3. These simulations were performed in spreadsheet using What’s Best 8.0 as the optimization tool, and the decision variables were set to assume binary values (1 and 0) based on a Branch and Bound method.

Consider a list of 8 non-identical three-state candidate components to be allocated in a series system, under distinct limitations of budget and number of components. The highlighted region of Table 5 brings the failure probabilities and costs of these components (randomly generated), while the first two columns from the left present size and budget restrictions for each simulation. The center of the same Table identifies the chosen components to become part of each system (1=in and 0=out), while the right side of the Table depicts the final reliability of such a system and its final cost. For instance, consider a system consisting of 4 components to be built under total cost of $25; the set of components 1, 2, 3, and 5 is the optimal solution, generating a final reliability of 0.9443 and costing $25.

### Table 5 - Allocations under distinct restrictions generated by the nonlinear optimization model (first set of $q_{io}$)

<table>
<thead>
<tr>
<th>Number of components</th>
<th>Max Budget</th>
<th>Comp.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Rel.</th>
<th>Final Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$q_o$</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
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</tr>
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</table>

Fonte: Autores

In order to evaluate the robustness of the optimization model, a second simulation was performed considering the same budget and number of components restrictions, but with higher failure probabilities and distinct costs from those presented in Table 5. The final results for this second set of simulations are depicted in Table 6, which follows the same organization of Table 5. The model performed consistently, spotting the best option in all tested scenarios (additional simulations regarding parallel systems are not presented due to space limitations).

### Table 6 - Allocations under distinct restrictions generated by the nonlinear optimization model (second set of $q_{io}$)

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<th>Max Budget</th>
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<th>7</th>
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<th>Rel.</th>
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Fonte: Autores

5. Conclusion

Evaluating the reliability of systems based on non-identical three-state components can be
extensively confusing and time consuming, since a highly combinatorial problem arises even from simple systems. This limitation leads to a lack of knowledge regarding the effect of distinct magnitudes of component failure probabilities over reliability profiles, especially when additional units are considered. This study presented a simplified approach to estimate the reliability of such systems based on mathematical recognition of patterns and implementation of such formulation in spreadsheet.

Reliability profiles of series and parallel systems were evaluated under distinct relations of open and short failure probabilities. Series systems are sensitive to variations on $q_{io}$, while parallel structures are strongly affected by modifications on $q_{is}$. The increasing number of components affected both series and parallel structures by reducing the reliability levels. This behavior is coherent for series systems; for parallel configurations, such result follows the theory of multi-state components structures, which states that there is an optimal number of redundant components to be used.

In addition, the integration of such formulation with an optimization solver proved to be robust and precise for indicating the best set of non-identical components for allocation in series and parallel systems, under limitations of budget and number of components.

References


