A Survey of Techniques for Optimizing Multiresponse Experiments

Flavio S. Fogliatto, Ph.D.
PPGEP / UFRGS
Praça Argentina, 9/ Sala 402 – Porto Alegre, RS 90040-020

Abstract

Most industrial processes and products are evaluated by more than one quality characteristic. To select the best design and operating control factors it is necessary to take into account all the measures of quality simultaneously, in what is usually referred to as multiresponse optimization. In this paper, we present a survey on multiresponse optimization techniques. We focus on the performance measures, such as deviation-from-target, variance, etc., considered in each technique. Since the underlying regression models for the responses are critical in terms of optimization accuracy, we also discuss typical regression procedures used in multiresponse optimization techniques, and classify those techniques according to their response modeling scheme.

Area: Engenharia do Produto / Projeto de Produto
Key Words: Multiresponse Optimization; Product Optimization; Design of Experiments.

1. Introduction

In many experiments it is important to evaluate the same experimental unit with respect to more than one response. Such experiments are known as multiresponse experiments (MREs), and are used in the development of most industrial processes and products where performance is dependent on more than one response output.

Optimizing an experiment usually implies determining a point in the design region at which the response presents some desired characteristics, most notably proximity to a specified value and small variance. This also holds of MREs, in which case all responses must be considered simultaneously and are unlikely to reach optimality at the same point.

Multiresponse optimization procedures are typically implemented in four steps:

(i) developing regression models individually for each response describing them as functions of the control factors;
(ii) computing predicted performance measures, such as distance-to-target and variance, using the predicted responses;
(iii) combining the predicted performance measures over all responses using some utility function or procedure; and
(iv) optimizing the function to identify the best settings for the control factors.

It is clear from the steps above that a proper modeling of responses is crucial if an optimization technique is to yield reliable results. A proper modeling of responses leads to low variance predictions. If the prediction is accurate, then observed responses
will be close to predicted values when the optimal levels of the control factors are retested in a confirmation trial.

Regression procedures typically applied for response modeling in the context of multiresponse optimization are OLS - ordinary least squares regression, GLS - generalized least squares regression, and MVR - multivariate regression [see Draper & Smith (1981), Myers (1986), and Seber (1984), respectively; see, also, Fogliatto & Albin (1998) for a thorough comparison of regression procedures in the light of multiresponse optimization].

In the OLS and GLS approaches, responses are modeled individually and are assumed to be uncorrelated. OLS also requires responses with homogeneous variances, which is not the case in GLS. OLS is used by the majority of authors dealing with MRE optimization, whether responses are correlated or not.

In the MVR approach, responses are modeled simultaneously and correlation among responses is considered. Furthermore, each response is modeled as a function of the same set of regressors but with different regression coefficients. MVR generates estimators of regression coefficients with variances smaller than those obtained by OLS and GLS if responses are indeed correlated. Similarly, GLS yields lower variance coefficients, if compared to OLS, when responses present heterogeneous variances.

In this work we present a survey of the recent literature on the analysis of multiresponse experiments. In the optimization techniques described next, the objective is to determine operating conditions at which responses simultaneously reach optimality, according to pre-defined criteria. A group of three optimization criteria are considered: (i) distance-to-target, measuring the deviation of a response outcome from a specified value; (ii) minimum variance; and (iii) robustness, measuring the sensitivity of a response to small variations in the settings of the control factors.

We next describe the techniques, their optimization criteria, and response modeling scheme.

2. Techniques for Multiresponse Optimization

Three major groups of optimization approaches can be identified in the literature. They are classified here according to their theoretical frameworks: the three groups are:

(i) response surface methodology related approaches;
(ii) approaches based on a utility function known as “desirability function”; and
(iii) approaches based on Taguchi (1986)’s Robust Design theory.

This last group is further divided into two categories: those based on Taguchi’s Signal-to-Noise performance measure, and those based on Taguchi’s Quadratic Loss Function. In the following paragraphs we identify the groups of approaches by shortly introducing them.

We now consider optimization methods based on response surface methodology. In the Contour Plot Optimization (CPO), we fit models to each response of an experiment and draw their contour plots, identifying the individual optima. By superimposing the plots, a common best region may be found. Distance-to-target is the optimization criterion considered. Limitations of CPO are given by the number of control factors, and the number of responses under study (optimization of experiments
with more than two responses using this method may be unmanageable). For application examples, see Lind et al. (1960), and Myers & Montgomery (1995).

Myers & Carter, Jr. (1973) developed a dual response system based on Ridge Analysis. Their objective is to maximize (minimize) one response, under the constraint that the other responses remain at certain targeted values. Using Lagrangian multipliers, the authors write regression models for both responses into a single objective function. The overall optimum is identified and analyzed using Ridge Analysis. An application of this same method is given in Biles (1975). In a variation of Myers & Carter’s method proposed by Del Castillo (1996), nonlinear programming is used to determine the overall optimum.

Myers et al. (1992) suggest alternative procedures to Taguchi’s Robust Parameter Design approach, based mainly on response surface methodology. They suggest the simultaneous modeling of the mean and variance of the responses, following the work of Box & Meyer (1986), and the determination of contour plots for these models. The optimum is identified through inspection of the plots, similar to the CPO approach previously presented, but with the response variance also considered as an optimization criterion.

Kim & Lin (1998) suggest a dual response surface optimization approach using fuzzy modeling. Membership functions for the variance and distance-to-target are developed to map the subjective goodness of design points; these functions are then used in the search for optimality. Their membership functions are indeed variations of Derringer & Suich (1980)’s desirability function, presented next.

The following references use the concept of a desirability function. Harrington (1965) introduces the desirability function by suggesting the calculation of desirability values associated to each outcome of an experiment. Outcomes positioned at the center of the operational region (which is limited by the upper and lower specification limits) are given a desirability value of 1.0. Conversely, outcomes outside those limits have desirability values equal to zero. Outcomes positioned at other points in the operational region are given values between 0 and 1. The geometric mean of the desirability outcomes for the different responses at each treatment is then used as a measure of the overall desirability, and related to the control factors by a regression model used to simultaneously optimize all different responses.

Derringer & Suich (1980) extended Harrington (1965)’s method to deal with responses that in addition to specification limits also have operational targets. They developed desirability expressions for three types of target: target is a value, target is zero, and target is infinity. Chang & Shivpuri (1994) use the desirability transformation as proposed by Derringer & Suich (1980) to collapse all outcomes into a single value, which is then used as the objective function in a nonlinear programming optimization, constrained by the design region. Del Castillo et al. (1996) modify Derringer & Suich (1980)’s desirability function to allow the assigning of importance weights to responses. Their modified function is differentiable at all points, being suitable for optimization using gradient-based optimization methods.

Fogliatto & Albin (1997A,B) use variations of Derringer & Suich (1980)’s function to create an utility function that handles three optimization criteria: distance-to-target, variance, and robustness of response outcomes. (The robustness term has been previously described for the single response case by Oh (1988).) The key idea to their method is to organize multiresponse experiments as hierarchies which are then optimized using analytic tools proposed by Saaty (1977) in the Analytic Hierarchy
Process. Their method allows for quantitative responses, responses based on expert’s opinion, and responses measured through sensory evaluation panels.

The next references are classified as Taguchi-based optimization approaches. In this paragraph, we describe those that use Taguchi’s Signal-to-Noise ratio as the performance measure. Methods based on Taguchi’s Quadratic Loss function are presented in the remaining paragraphs. León et al (1987) suggests the use of PerMIAs (Performance Measures Independent of Adjustment) as the objective function in a single-response optimization procedure, with distance-to-target and variance as the optimization criteria. The PerMIAs are, indeed, variations of Taguchi’s signal-to-noise (S/N) ratio. [For a comprehensive explanation on S/N ratios, see Box (1988).] The authors show that in some situations, Taguchi’s S/N Ratio and his Quadratic Loss Function lead to the same result. The same approach was later extended to the multiresponse case by Elsayed and Chen (1993). Logothetis & Haigh (1988) present two performance statistics based on Taguchi’s S/N ratio to optimize multiresponse experiments. Their emphasis, though, is on determining data transformations that lead to a proper use of their (and, consequently, Taguchi’s) performance statistics.

Khuri & Conlon (1981)’s “generalized distance” approach searches for treatments where the squared distance of the outcomes from their targets is minimized. Their utility function is a variation of Taguchi’s Quadratic Loss function. In their procedure, targets are treated as random variables rather than fixed values, which is particularly suitable to situations where the responses’ regression models present severe lack-of-fit.

Tribus & Szonyl (1989) seem to be the first to explicitly suggest the use of Taguchi’s Quadratic Loss Function as a performance statistic in multiresponse optimization. More precisely, their performance statistic is given by the expected value of Taguchi’s Loss Function. Distance-to-target and variance are the optimization criteria considered. All Taguchi-based approaches mentioned so far are presented and compared in Yum & Ko (1991).

Raiman & Case (1992) use a Loss Function similar to that developed by Tribus & Szonyl (1989) to simultaneously optimize several responses. Their emphasis, though, is in the determination of proper cost coefficients to be used in the function. Pignatiello, Jr. (1993) uses the expected value of Taguchi’s Quadratic Loss Function as the objective function in his optimization procedure. The distance-to-target and the variance of the responses, weighted by cost coefficients, are considered in the search for the best set of control factors. The optimization is carried out by determining the expected loss at each treatment, and building a model where the expected loss is a function of the x’s (vector of settings for the control factors). The best setting for the x’s is the one that minimizes the expected loss.

A loss function similar to the one in Pignatiello, Jr. (1993) is presented by Ribeiro & Elsayed (1995) with the addition of a robustness term. In addition to searching for treatments yielding responses close to target and with small variance, the authors look for settings where the variance of the control factors (treated thus as random variables) have little effect on the overall variance of the responses. León (1996) adds specification limits to the loss function; distance-to-target is the only optimization criterion considered.

Some of the works above have been outlined in book chapters on multiresponse optimization; see Khuri (1990), Myers and Montgomery (1995), and Khuri & Cornell (1996).

In all but two of the methods described previously, responses are modeled individually using Ordinary Least Squares regression. We verified however that OLS assumptions were not always satisfied. In Ribeiro & Elsayed (1995) and Derringer & Suich (1980), for example, responses presented correlation as high as 0.8: OLS is obviously a poor choice of modeling procedure in these cases. In addition, most references where minimum variance was an optimization criterion presented responses with heterogeneous variances, and were still erroneously modeled using OLS.


The multiresponse optimization techniques surveyed above are organized according to their theoretical framework and summarized in Figure 1.

3. Conclusion

Experimental units are often evaluated with respect to more than one quality characteristic. When optimizing such multiresponse experiments, the objective is to select the control factor settings to achieve best responses across all performance measures. Typical performance measures are distance-to-target, variance and robustness.

A survey on past and recent approaches for the optimization of multiresponse experiments is presented. Optimization techniques are classified according to their common characteristics, such as optimality criteria and theoretical framework. The regression procedures used for response modeling in each optimization technique are also identified and discussed.

4. References


# Approaches for Multi-Response Optimization

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Figure 1. Multiresponse optimization techniques grouped according to their theoretical framework.


